# **Appendix B** : Natural Monopoly and the AFN

## I. Competitive Market



Under perfect competition, the quantity supplied equals the quantity demanded, Q\*, and price is symmetric: the price to the consumer equals the price to the producer: The price to the producer at equilibrium includes a "normal profit." A normal profit is a profit attractive enough to permit this supplier to remain in business, but no more.

#### II. The Firm at Equilibrium



All firms in a market are "price takers:" no firm is large enough to significantly shift the supply curve for the market. That is, each firm treats the market price (P\*, determined in Graph 1) as its demand curve and as a given. A demand curve for the firm, then, can be represented by the horizontal line at the equilibrium price, P\*, established in figure 1. At equilibrium, this horizontal line will pass through the minimum average cost point for the firm.

For any given quantity Q, Average Cost is determined as the sum of marginal cost up to that Quantity divided by Q:

$$AC = \sum_{i} \frac{MC_i}{Q}$$

Therefore AC increases or decreases depending on whether MC is greater than or less than AC at that particular quantity-point.

printed 9/3/2003: 11:40 AM



In a market in which the total quantity demanded can be supplied by a single firm whose marginal cost is everywhere less than its average costs constitutes a natural monopoly. That is, the product can be supplied to the market at a cost that is everywhere declining.

Should a second firm enter in competition with the initial firm, the cost to society would rise; each firm would face a lesser demand, and each firm's output would occur at a higher average cost than that possible to the single firm supplying total demand.

Under a Natural Monopoly, the firm would not cover its costs were it to set  $P^* = MC$ . If left free to maximize its profit ( $\Pi$ ), it would do so by setting MR = MC. This would occur at  $Q_m^*$  and at a price of  $P_m^*$  as shown in the figure.

(Why? See Appendix A)

### **IV. Pricing the AFN**



An AFN market is an instance of Natural Monopoly for which Marginal Cost < AC everywhere, as suggested in this figure.

**Pricing is asymmetric**: The price is to the user is different from the price that must be paid to the producer. The price to the user is zero; the effective price to the producer must be high enough to get the AFN produced in the first place.

$$P_{\mu} = MC = 0$$

$$P_p = P_p = High Enough$$

## **Equations for Optimizing Monopoly Profit**

In a monopoly situation, the monopolist's goal is to maximize profit and he does so by choosing to produce only the quantity for which marginal cost will be equal to the marginal revenue for that output.

The reason for this is detailed in this appendix.

$$\pi = R - C \tag{1}$$

where  $\pi$  is profit, R is revenue, C is total cost and Q is quantity. The maximum point for profit is at:

$$\frac{d\pi}{dQ} = 0; \qquad \text{i.e.} \qquad \frac{dR}{dQ} - \frac{dC}{dQ} = 0; \qquad (2)$$

or:

$$\frac{dR}{dQ} = \frac{dC}{dQ}; \tag{3}$$

the quantity at which marginal cost is equal to marginal revenue; MR = MC. The equations for deriving the marginal revenue (MR) curve are as follows:

Assuming that the demand curve is a straight line crossing the Price axis at  $P_0$  and the Quantity axis at  $Q_0$ , the equation for the demand line (or price line) is then,

$$D = P_0 - \left[\frac{P_0}{Q_0}\right] Q \tag{4}$$

Revenue is equal to price times quantity:

$$R = P * Q = \left( P_0 - \left[ \frac{P_0}{Q_0} \right] \right) * Q = P_0 * Q - \left[ \frac{P_0}{Q_0} \right] * Q^2$$
(5)

Marginal revenue is the first derivative of revenue:

$$MR = \frac{dR}{dQ} \tag{6}$$

$$MR = P_0 - 2\left(\frac{P_o}{Q_o}\right) * Q \tag{7}$$

This equation is similar to the equation for the demand curve (or price line), except that the slope is twice as steep. Therefore, it intercepts the Price axis at P<sub>0</sub>, and the Quantity axis at  $(1/2)*Q_0$ , which implies that it bisects the angle that the demand line makes with the price axis. The marginal cost (MC) curve is the supply curve for the monopolist in diagram 3. MC = MR occurs at the intersection between these two curves.